II-2 A FREQUENCY TRANSFORMATION FOR COMMENSURATE TRANSMISSION-LINE NETWORKS*

E.G. Cristal

Stanford Research Institute

Frequency transformations are commonly used in lumped-element network theory to convert a given filter network into a related filter network. For example, an often used frequency transformation is

$$s' \rightarrow As \quad , \qquad \text{(II-1)}$$

where the symbol → stands for "is replaced by," A is a constant, the primed variable is that of the original network, and the unprimed variable is that of the transformed network.** Transformation (II-1) is used to scale the bandwidth of the existing network to another preferred value. Other commonly used frequency transformations in lumped filter theory are

$$s' \rightarrow A/s \quad , \qquad \text{(lowpass to highpass transformation)} \qquad \text{(II-2)}$$

$$s' \rightarrow \omega\left(\frac{s}{\omega_0}\right)+\left(\frac{\omega_0}{s}\right) \qquad \text{(lowpass to bandpass transformation)} \qquad \text{(II-3)}$$

$$s' \rightarrow \frac{\quad}{\omega\left(\frac{s}{\omega_0}\right)+\left(\frac{\omega_0}{s}\right)} \qquad \text{(lowpass to bandstop transformation)} \qquad \text{(II-4)}$$

It is emphasized that in all cases the usefulness of these transformations lies in the fact that their effects on the responses of the network are easily related to changes in the element values of the network. Because such frequency transformations are available, a given lowpass filter may function as a prototype for a number of different types of filters, obviating the compilation of a multitude of designs for lowpass, highpass, bandpass, and bandstop filters.

Analogous transformations would be equally useful for commensurate transmission-line networks, if they could be developed. For the special class of commensurate transmission-line networks consisting of open- and short-circuited stubs, ideal transformers, and resistors, but without unit elements (i.e., quarter-wavelength lines), transformations (II-1) through (II-4) can indeed by used. However, in most cases realizations of commensurate transmission-line networks without unit elements is impractical or impossible. Unfortunately, in the more general case of commensurate transmission-line networks, consisting of open- and short- circuited stubs, ideal transformers, resistors, and unit elements, the usual frequency transformations of lumped-element network theory cannot be used. The fundamental reason for this is that for transmission-line networks with unit elements, the effects of the transformations on the network responses are not easily (if at all) related to changes in the element values of the network. However, an exception to this statement is the frequency transformation.*

$$S' \rightarrow 1/S \qquad \text{(II-5)}$$

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**Throughout this paper we shall use primed variables to represent parameters of the original network and unprimed variables for those in the transformed network.

It has been found that for this transformation the effects on the network elements can be relatively easily accounted for. This transformation corresponds to Eq. (II-2), with A = 1, and possesses corresponding properties. However, its effects on the parameters of transmission-line networks can be substantially different from the effects on the parameters of lumped-element networks because of the existence of unit elements. It is easily shown that transformation (II-5) takes transmission-line filters into transmission-line transformers and vice versa, low-pass distributed filters into high-pass distributed filters and vice versa, elliptic-function bandstop distributed filters into elliptic-function bandpass distributed filters and vice versa. Sections II-B through D of this paper investigate the properties of transformation (II-5) and describe the relationships between the original and transformed networks. The last section explores possible applications to which the transformation may be put.

*For commensurate transmission-line networks, the variable S' represents tanh (γL), where γ is the complex propagation constant and L is the commensurate length of the transmission lines.